

## THE WAVE RESISTANCE OF AMPHIBIAN AIRCUSHION VEHICLES IN BROKEN ICE

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UDC 624.124; 532.595

The ice-breaking properties of amphibian air cushion vehicles (AACV), which have recently been discovered, [1] make it necessary to solve a number of new applied problems [2]. One of the promising methods of ice breaking with the aid of AACV is the resonance method [2] which is applied at speeds corresponding to maximum wave resistance. In this connection determination of the wave resistance to AACV under ice conditions becomes very important. In the absence of ice this problem has been theoretically solved for the movement of a vehicle in deep and shallow water [3, 4], in a channel [5], and with acceleration [6]. The present paper is concerned with the stationary problem of the wave resistance to AACV in broken ice.

1. Let there be a given system of the AACV surface pressures moving at a constant velocity  $u$  over an infinite water field covered by broken ice. In accordance with the principle of inverse motion, we assume that a load  $q(x, y)$  is applied to the free liquid surface covered by broken ice and moving with velocity  $-u$  as  $x \rightarrow \infty$ . The coordinate system that is stationary relative to the vehicle is located as follows: the plane  $xOy$  coincides with the unperturbed ice-water interface, the  $x$  axis points in the direction of the vehicle's motion, and the  $z$  axis points vertically upward. The water is assumed to be an ideal incompressible liquid with density  $\rho_2$ . Broken ice is represented in the form of floating disconnected masses. Interaction forces between separate ice floes are ignored, and their dimensions are considered sufficiently small compared with the wavelength so that ice-floe bending does not occur [7]. Full-scale tests [2] show this approach to be quite justifiable in solving problems on the propulsive properties of AACV in ice broken by the resonance method.

Use is made of the assumption that the field covered by broken ice is continuous [7], and the surface density coinciding with the floating-particle mass per unit area is given by the continuous function

$$m(x, y) = \rho_1 h \equiv \rho_1^0 s(x, y) h(x, y), \quad (1.1)$$

where  $\rho_1^0$  is the ice physical density;  $s(x, y)$  is a dimensionless function of ice-floe tightness [7] ( $0 \leq s \leq 1$ ); and  $h(x, y)$  is the ice thickness. To simplify the problem, the quantities  $h$  and  $s$  are further considered constant.

In the adopted coordinate system, the velocity potential  $\varphi(x, y, z)$  of fluid perturbed motion must satisfy the Laplace equation  $\Delta\varphi = 0$  and the linearized boundary conditions

$$z = 0: \frac{\partial^2 \varphi}{\partial x^2} - \mu \frac{\partial \varphi}{\partial x} + \frac{g}{u^2} \frac{\partial \varphi}{\partial z} + \frac{\rho_1 h}{\rho_2} \frac{\partial^3 \varphi}{\partial z \partial x^2} = \frac{1}{\rho_2 u} \frac{\partial q}{\partial x}, \quad z = -H: \frac{\partial \varphi}{\partial z} = 0. \quad (1.2)$$

Here  $\mu > 0$  is the coefficient of scattering forces [3, 8];  $H = H_1 - a$ ;  $H_1$  is the water-body depth; and  $a = h\rho_1^0/\rho_2$  is the ice immersion depth at static equilibrium. For great depths, when  $H_1 \gg h$ , it can be assumed that  $H \approx H_1$ .

According to [3, 9], the wave resistance to AACV is numerically equal to the horizontal projection of the resultant of pressure forces onto the surface

$$R = \iint_{(\Omega)} q(x, y) \frac{\partial w(x, y)}{\partial x} dx dy, \quad (1.3)$$

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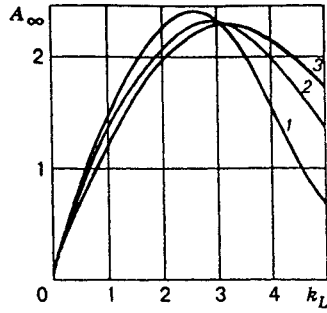


Fig. 1

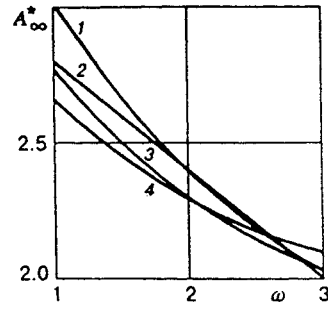


Fig. 2

where  $\Omega$  is the domain of load distribution  $q(x, y)$  and  $w(x, y)$  is the floating-fluid surface deformation defined in the linear theory of waves as [10]

$$w(x, y) = \frac{u}{g} \left( \frac{\partial \varphi(x, y, z)}{\partial x} \right) \Big|_{z=0} - \frac{q(x, y)}{\rho_2 g} + \frac{u \rho_1 h}{\rho_2 g} \left( \frac{\partial^2 \varphi(x, y, z)}{\partial x \partial z} \right) \Big|_{z=0}. \tag{1.4}$$

The desired potential  $\varphi$  is calculated by the scheme suggested in [3]. According to [3, 9], the functions  $\varphi(x, y, z)$  and  $q(x, y)$  are written in the form of Fourier integrals

$$\begin{aligned} \varphi(x, y, z) &= \frac{1}{4\pi^2} \int_0^\infty k dk \int_{-\pi}^\pi d\theta \iint_{(\Omega)} (Ae^{-kz} + Be^{kz}) \exp\{ik[(x - x_0) \cos \theta + (y - y_0) \sin \theta]\} dx_0 dy_0, \\ q(x, y) &= \frac{1}{4\pi^2} \int_0^\infty k dk \int_{-\pi}^\pi d\theta \iint_{(\Omega)} q(x_0, y_0) \exp\{ik[(x - x_0) \cos \theta + (y - y_0) \sin \theta]\} dx_0 dy_0. \end{aligned} \tag{1.5}$$

Here  $A$  and  $B$  are unknown functions of the variables  $x_0, y_0, k$ , and  $\theta$ , which are to be determined.

Substitution of (1.5) into boundary conditions (1.2), application of the residue theory and subsequent passage to the limit at  $\mu \rightarrow 0$  [3, 8] allow one to obtain the potential function  $\varphi(x, y, z)$ . The wave resistance of the system of surface pressures is shown to be determined only by the part of the potential that at subcritical motion velocities ( $u < \sqrt{gH}$ ) has the form

$$\begin{aligned} \varphi(x, y, z) &= -\frac{1}{2\pi \rho_2 u} \iint_{(\Omega)} q(x_0, y_0) dx_0 dy_0 \int_{\lambda_0}^\infty \frac{\cosh(\lambda(z + H))}{\cosh(\lambda H)} \\ &\times \frac{\cos \left[ (x - x_0) \sqrt{\nu \lambda \tanh(\lambda H) / (1 + \rho_1 h \lambda \tanh(\lambda H) / \rho_2)} \right]}{\sqrt{\lambda^2 - \nu \lambda \tanh(\lambda H) / (1 + \rho_1 h \lambda \tanh(\lambda H) / \rho_2)} (1 + (\rho_1 / \rho_2) h \lambda \tanh(\lambda H))} \\ &\times \cos \left[ (y - y_0) \sqrt{\lambda^2 - \nu \lambda \tanh(\lambda H) / (1 + \rho_1 h \lambda \tanh(\lambda H) / \rho_2)} \right] \lambda d\lambda, \end{aligned} \tag{1.6}$$

where  $\nu = g/u^2$  and  $\lambda_0$  is a solution of the transcendental equation  $\nu \tanh(\lambda H) = \lambda (1 + (\rho_1 / \rho_2) h \lambda \tanh(\lambda H))$ .

At critical and supercritical motion velocities ( $u \geq \sqrt{gH}$ ) in expression (1.6) for the potential, the quantity  $\lambda_0$  is replaced by zero.

2. In the case of an infinitely deep liquid body ( $H = \infty$ ) relation (1.6) is simplified. In terms of (1.3) and (1.4) the formula for the wave resistance of the rectangular system of constant pressures [ $q(x, y) = q_0 \equiv \text{const}$ ] moving over the broken ice surface takes the form

$$R_\infty / D = A_\infty q_0 / (\rho_2 g L). \tag{2.1}$$

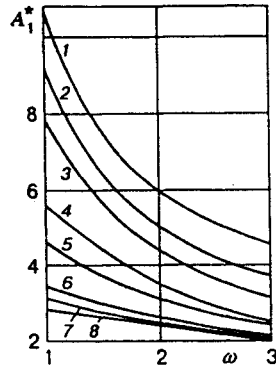


Fig. 3

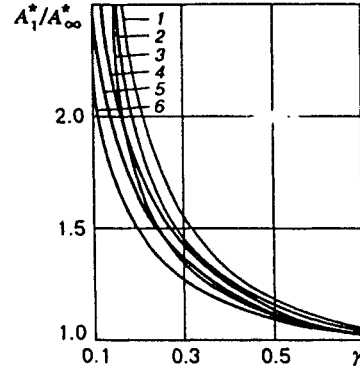


Fig. 4

Here

$$A_{\infty}(k_L, \omega, \alpha) = \frac{8\omega}{\pi} \int_{\beta}^{\infty} \sin^2 \left( \frac{1}{2} \sqrt{\frac{k_L \lambda}{1 + \alpha \lambda}} \right) \sin^2 \left( \frac{1}{2\omega} \sqrt{\lambda^2 - \frac{k_L \lambda}{1 + \alpha \lambda}} \right) \left( \lambda^2 - \frac{k_L \lambda}{1 + \alpha \lambda} \right)^{-3/2} \lambda d\lambda;$$

$k_L = gL/u^2$ ;  $D = q_0LB$ ;  $L$  and  $B$  are the length and width of the aircushion;  $\omega = L/B$  is its aspect ratio;  $\alpha = \rho_1 h/(\rho_2 L)$  is a dimensionless parameter that characterizes the ice tightness and thickness; and  $\beta$  is a solution of the quadratic equation  $\lambda(1 + \alpha\lambda) = k_L$ .

It should be noted that in the limiting case where  $h \rightarrow 0$  or  $\rho_1 \rightarrow 0$  ( $s \rightarrow 0$ ), formula (2.1) becomes the relation obtained in [3] for the "clean" surface of an infinitely deep liquid body.

The main results of the numerical calculations by formula (2.1) are presented in Figs. 1 and 2. Curves 1-3 in Fig. 1, which correspond to  $\omega = 2$  and  $\alpha = 0, 0.045$ , and  $0.09$ , illustrate the influence of broken ice on the wave-resistance coefficient as a function of  $k_L$ . It can be seen that as  $\alpha$  grows, the point of absolute maximum of the coefficient  $A_{\infty}$  is shifted toward lower velocities, and the absolute maximum decreases. Curves 1, 3, and 4 in Fig. 2 reflect the dependence of the absolute maximum of the wave resistance coefficient  $A_{\infty}^*$  on the parameter  $\omega$  for  $\alpha = 0, 0.045$ , and  $0.09$ , respectively. Analysis of the behavior of the curves suggests that when AACV move over the surface of an infinitely deep water body the effect of broken ice on the wave resistance of the vehicle is insignificant. For  $\alpha \leq 0.09$  and  $\omega = 1.5-3.0$ , the absolute maximum of the wave-resistance coefficient can be calculated with an error smaller than 7% from the linear dependence below (straight line 2 in Fig. 2):

$$A_{\infty}^* = 3.2 - 0.4\omega. \quad (2.2)$$

3. We consider the motion of the rectangular system of constant pressures  $q_0$  over the surface of a finite-depth water body. In the case of subcritical velocities ( $u < \sqrt{gH}$ ), taking formulas (1.3)-(1.5) into account leads to the following relation for the wave resistance of the rectangular system of constant pressures  $q_0$  in broken ice:

$$R/D = q_0 A_1 / (\rho_2 g L). \quad (3.1)$$

Here

$$A_1(k_H, \omega, \gamma, \alpha) = \frac{8\omega\gamma}{\pi} \int_{\tau_0}^{\infty} \sin^2 \left( \frac{1}{2\gamma} \sqrt{\frac{k_H \tau \tanh \tau}{1 + \alpha \gamma^{-1} \tau \tanh \tau}} \right) \times \sin^2 \left( \frac{1}{2\omega\gamma} \sqrt{\tau^2 - \frac{k_H \tau \tanh \tau}{1 + \alpha \gamma^{-1} \tau \tanh \tau}} \right) \left( \tau^2 - \frac{k_H \tau \tanh \tau}{1 + \alpha \gamma^{-1} \tau \tanh \tau} \right)^{-3/2} \tau d\tau;$$

$\gamma = H/L$ ;  $k_H = gH/u^2$ ; and  $\tau_0$  is a solution of the transcendental equation  $k_H \tanh \tau = \tau(1 + \alpha \gamma^{-1} \tau \tanh \tau)$ .

At supercritical velocities of motion ( $u \geq \sqrt{gH}$ ), formula (3.1) remains valid, provided that  $\tau_0 = 0$ .

The results of the numerical calculation of formula (3.1) are presented in Figs. 3 and 4. Figure 3 shows the dependence of  $A_1^*$  on the aspect ratio  $\omega$  for various  $\alpha$  and  $\gamma$ . Curves 1–3 correspond to  $\gamma = 0.15$  and  $\alpha = 0, 0.018,$  and  $0.045$ ; curves 4 and 5 to  $\gamma = 0.3$  and  $\alpha = 0$  and  $0.045$ ; curves 6 and 7 to  $\gamma = 0.6$  and  $\alpha = 0$  and  $0.045$ ; and curve 8 is relation (2.2). It is seen that with decreasing relative depth  $\gamma$  the effect of the parameters  $\alpha$  and  $\omega$  on  $A_1^*$  increases.

Figure 4 presents the relative quantity  $A_1^*/A_\infty^*$  versus  $\gamma$ , where the coefficients  $A_1^*$  and  $A_\infty^*$  are taken at equal  $\alpha$  and  $\omega$ . Curves 1–3 correspond to  $\alpha = 0$  and  $\omega = 1.67, 2,$  and  $2.5$ ; curves 4–6 to  $\alpha = 0.045,$  and  $\omega = 1.67, 2,$  and  $2.5$ . It can be seen that with decreasing depth  $A_1^*$  sharply increases and substantially exceeds  $A_\infty^*$ . With increasing  $\gamma$  the value of  $A_1^*/A_\infty^*$  tends to unity.

The calculations performed show that the wave resistance of AACV in broken ice for a relative water-body depth  $\gamma \geq 0.8$  can be calculated by formula (2.2). As the water-body depth decreases, the absolute maximum of the wave-resistance coefficient increases sharply, and the influence of the broken ice is enhanced as well.

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